Nuclear Structure Theory II
The Nuclear Many-body Problem

Alexander Volya
Florida State University
Physics of light nuclei

Chart of light nuclei colored by lifetimes

- $\tau_{1/2} < 0.1$ as
- $0.1$ as $< \tau_{1/2} < 1$ ms
- $1$ ms $< \tau_{1/2} < 0.1$ s
- $0.1$ s $< \tau_{1/2} < 3$ s
- $3$ s $< \tau_{1/2} < 2$ min
- $2$ min $< \tau_{1/2} < 1$ hour
- $1$ day $< \tau_{1/2} < 1$ y
- $1$ y $< \tau_{1/2} < 10$ Gy
- $\tau_{1/2} > 1$ Gy
Physics of deuteron, $A=2$

Nucleon-nucleon interaction
## Physics of deuteron, $A=2$

**Symmetry, isospin and two-nucleon states**

### Notation:

\[ \frac{2S+1}{(\ell)}_J \]

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<tr>
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<th>(J = l)</th>
<th>(J = l - 1)</th>
<th>(J = l)</th>
<th>(J = l + 1)</th>
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<td>1(d_2)</td>
<td>1(f_3)</td>
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<td>1(g_4)</td>
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What are deuteron’s quantum numbers?

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<th>0(^+)</th>
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<td>Triplets</td>
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Physics of deuteron, $A=2$
Symmetry, isospin and two-nucleon states

Argonne $V_4$ potential

Prediction for tritium ($^3\text{H}$) is 7.65 MeV binding
Level scheme of light nuclei
Halo nuclei, $^{11}\text{Be}$

**Extended radius**

**Halo density**
Clustering in light nuclei

\[
\begin{align*}
\alpha + p & \rightarrow ^3\text{Li}_2, \quad E = -26.33 \\
\alpha + n & \rightarrow ^5\text{He}_3, \quad E = -27.40 \\
\alpha + n & \rightarrow ^5\text{Li}_2, \quad E = -26.33 \\
\alpha + n & \rightarrow ^6\text{He}_4, \quad E = -29.268 \\
\alpha + n + n & \rightarrow ^{10}\text{Be}_6, \quad E = -65.0 \\
\alpha + n + n + \alpha & \rightarrow ^{16}\text{O}_8, \quad E = -127.6 \\
\end{align*}
\]
Chart of Isotopes
Nuclear many-body problem
configuration interaction, the shell model

- Many-body configurations and Hamiltonian
- Example study
- Binding energy, shell evolution and monopole
- Pairing interaction
- Multipole-multipole interaction, emergence of deformation and rotations
- Statistical approach and random matrix theory
The Nuclear Shell Model

Many-body Hamiltonian

\[ H = \sum_a \frac{\vec{p}_a^2}{2m} + \sum_{a>b} V_{NN}(r_a - r_b) \]

Mean field and residual interactions

\[ H = \sum_a \left[ \frac{\vec{p}_a^2}{2m} + U(r_a) \right] + \sum_{a>b} V_{NN}(r_a - r_b) - \sum_a U(r_a) \]

Residual interactions

- Residual, depends on mean-field
- Depends on truncation of space and basis
- Not exactly two-body
Many-body formalism

Many-body state

\[ |\{n_\lambda\}\rangle = |n_1, n_2, \ldots, n_\lambda, \ldots\rangle \]

Creation and annihilation operators

\[ a_\lambda | \ldots, n_\lambda, \ldots \rangle = (-)^{n_\lambda} \sqrt{n_\lambda} | \ldots, n_\lambda - 1, \ldots \rangle, \]
\[ a_\lambda^\dagger | \ldots, n_\lambda, \ldots \rangle = (-)^{n_\lambda} \sqrt{1 - n_\lambda} | \ldots, n_\lambda + 1, \ldots \rangle \]

\[ H' = \frac{1}{2} \sum_{a \neq b} U_{ab} \]
\[ = \frac{1}{2} \sum_{1234} (12|U|34) a_1^\dagger a_2^\dagger a_3 a_4 \]
Two-body Hamiltonian in the particle-particle channel:

\[ H = \sum_L V_L \sum_\Lambda P^\dagger_{L\Lambda} P_{L\Lambda} \]

where

\[ P^\dagger_{L\Lambda} \propto (a_1^\dagger a_2^\dagger)_{L\Lambda} \]

No-core shell model
- bare NN interaction
- Renormalized interactions improve convergence

Traditional shell model
- Simple potential interactions
- Renormalized bare interactions to include core and core excitations
- Phenomenological interactions determined from fits.
Typical shell model study

Shell model codes:
NuShell: http://knollhouse.eu/NuShellX.aspx
Redstick: http://www.phys.lsu.edu/faculty/cjohnson/redstick.html
CoSMo: http://www.volya.net (click CoSMo)
For demonstration we use CoSMo code.

Anatomy of a shell model study
1. Identify system, valence space, limitations on many-body states to study (cosmoxml)
2. Create a list of many-body states, typically fixed $J_z$ projection $T_z$, and parity (Xsysmbs)
3. Create many-body Hamiltonian (XHH+JJ)
4. Diagonalize many-body Hamiltonian using exact, lanczos, davidson or other method (texactev, davidson_file).
5. Database eigenstates states and determine their spins (XSHLJT)
6. Define other operators and compute various properties
   • Overlaps and spectroscopic factors (XSHLSF)
   • Electromagnetic transition rates (XSHLEMB)
The simple model

• Single-\( j \) level
  • \( \Omega = 2j + 1 \) single-particle orbitals: \( m = -j, j-1, \ldots, j \)
• Number of nucleons \( N \): \( 0 \leq N \leq \Omega \)
• Number of many-body states: \( \Omega \!/(N!(\Omega-N))! \)
• Many-body states classified by rotational symmetry: \( (J,M) \)

Dynamics

• Rotational invariance and two-body interactions
  particle-particle pair operator \( P_{LM} = (a\ a)_{LM} \)
  particle-hole pair operator \( M_{K\kappa} = (a\ a^\dagger)_{K\kappa} \)

\[
H = \sum_L V_L \sum_M P^\dagger_{LM} P_{LM}
\]

• Hamiltonian

• Dynamics is fully determined by \( j+1/2 \) parameters \( V_L \)
To the lowest order binding energy is proportional to the number of nucleons.

\[ E(N) \sim \epsilon N \]
N=28 Isotones, data

1-particle $J=j=7/2$

2 particles $J=0,1,2,...,7$
but Pauli principle $J=0,2,4,6$

3 particles
Total states $56=8!/(5!3!)$

$J=15/2,11/2,9/2,7/2,5/2,3/2$

3-particles on $j=7/2$, 28 $m>0$ states

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Consider a constant component “shift”

“Shift term” counts number of pairs

What is needed to fix closed shell $^{56}\text{Ni}$ ?

Fully occupied shell $^{56}\text{Ni}$

Monopole term

Matching binding across the shell gives

$$H = \epsilon N + \sum_{L=0,2,4,6} V_L \sum_{M=-L}^{L} P_{LM}^\dagger P_{LM}$$
N=28 isotones Monopole term

Prediction $S_p=7.374$ MeV in $^{56}$Ni

The single-particle energy changes from 9.6 MeV to 7.3 MeV

Woods-Saxon prediction 9.9 MeV to 7.2 MeV

$$E - \epsilon N - N(N-1)\tilde{V}_0/2$$

$$\epsilon = -9.626 \text{ MeV} \quad \tilde{V}_0 = 0.3217 \text{ MeV}$$
Shell evolutions and monopole term

Energy:

\[ E(N) \sim \epsilon N + N(N - 1)\tilde{V}_0/2 \]

\[ \epsilon = \frac{\partial E}{\partial N} \sim \epsilon + \tilde{V}_0 N \]

Effective single-particle energies

Tensor nucleon-nucleon interaction
off-diagonal monopole term

\[ \epsilon_i = \epsilon_i + \sum \tilde{V}_0^{(i,j)} N_j \]

Example from Otsuka, GXPF1 interaction
Proton energy in f7/2 shell
N=28 Best fit

Overall spectrum, ordering is well reproduced 31 state only 5 parameters
There are discrepancies, p-h symmetry, seniority

<table>
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<th>$\epsilon$</th>
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<td>$V^{(2)}_0$</td>
<td>$-9827(16)$</td>
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<td>$V^{(2)}_2$</td>
<td>$-2033(60)$</td>
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<td>$V^{(2)}_4$</td>
<td>$-587(39)$</td>
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<td>$V^{(2)}_6$</td>
<td>$443(25)$</td>
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<td>$V^{(2)}_8$</td>
<td>$887(20)$</td>
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Two-body interaction
pairing and potential model

Phenomenological residual interaction

\[ V(r_1 - r_2) \sim \delta^3(r_1 - r_2) \]

\[ V_L \sim \frac{1}{2L + 1} \left| C_{j1/2}^{L0} \right|^2 \]

Short range interaction contributes most to pairing matrix element \( V_0 \)
Pairing interaction in f7/2 shell nuclei
Pairing interaction in nuclei

\[ \Delta = 12A^{-1/2} \]

- N=8
- 20
- 28
- 50
- 82
- 126

mass number \( A \)
Pairing interaction in nuclei
Literature

- D. Dean et al. Progress in Particle and Nuclear Physics 53 (2004) 419–500
- Broglia, Zelevinsky (eds), Fifty years of nuclear BCS, Pairing in Finite Systems. (World Scientific, 2013)
- Rowe, Nuclear Collective Motion Models and Theory, (World Scientific, 2010)